PART ONE

Agriculture as a Sector

2

The Economy under Constant Resources and Technology

This chapter lays out the basic analytic framework for evaluating the relationships between agriculture and the rest of the economy. The model is based on strong assumptions that are removed as the discussion progresses, but nevertheless it captures some of the important features for understanding the main issues associated with agricultural growth.

It is assumed that the economy consists of two sectors, agriculture and nonagriculture. Each sector uses two factors of production, labor and capital, whose supply is fixed. In addition, agricultural production uses land. Each sector is characterized by competitive conditions and full employment. Technology does not change. The demand specification is general; in particular, the underlying utility function is not restricted to be homothetic.

This simple framework introduces concepts and results that are important for the subsequent discussion: the sectoral supply function, the relationship between the product and factor markets, the shadow price of the fixed factors of production, and the presentation of the equilibrium solution in a form that highlights the interdependence of these results. The link between factor and product markets depends on factor intensity; thus a discussion of the measurement of factor intensity is essential. At the end of the chapter, two other aspects related to the application of this model to agriculture are discussed: the introduction of land as a factor of production and a differentiation between food and agriculture. It is shown that with appropriate interpretation, the framework developed here accommodates these extensions

Supply

Assumptions

The economy consists of agriculture (sector 1) and nonagriculture (sector 2), and two factors, capital (K) and labor (L).

Admissible allocation: The total sectoral use of the two factors does not exceed their total supply.

$$K \geq K_1 + K_2 \tag{2.1}$$

$$L \geq L_1 + L_2 \tag{2.2}$$

Technology: The technology is represented by concave, twice differentiable, constant returns to scale (CRS) production functions:[1]

$$Y_i = F_i(K_i, L_i), \quad i = 1, 2, \tag{2.3}$$

where Y_i is the output of sector i.

Competition: Perfect competition exists among firms and consumers within each sector and between sectors. Consequently, the value marginal productivities are equal to their respective factor prices. Let p_i, w, and r be the accounting price of product i, the wage, and the rental rates, respectively, then

$$r = p_i \frac{\partial F_i}{\partial K_i} \tag{2.4}$$

$$w = p_i \frac{\partial F_i}{\partial L_i} \tag{2.5}$$

The micro underpinnings of the model will not be discussed in detail until later. The assumptions of perfect competition and of CRS, however, raise the question of how the size of the firm is determined. This question can be answered in various ways, including uncertainty and cost of adjustment. These, however, require an extension of the framework that will complicate the discussion. Instead, it is assumed, for now, that the sectoral output is determined by the exit and entry of firms. For this purpose it is assumed that the production function of each firm contains a variable representing the entrepreneurial or managerial capacity of the firm. The production function is linear homogeneous in all the inputs including entrepreneurial capacity, which at this stage is assumed to be fixed for the firm. It is assumed that the supply of entrepreneurs is perfectly elastic, which results in zero profits.

The Structure of Supply

The eight relations in (2.1)–(2.5) summarize the supply conditions under constant technology and fixed factor supply. The endogenous variables in the economy are L_i, K_i, Y_i, w, r, and p_i. The system has no money, and therefore it determines only the product price ratio $p = p_1/p_2$. With equations

(2.1)–(2.5), the supply side can be summarized by solving the system in terms of one of the endogenous variables.

For convenience, the system is written in more compact form. In this, we assume full employment conditions so that (2.1) and (2.2) are combined to yield

$$k = \ell_1 k_1 + \ell_2 k_2 \tag{2.6}$$

$$1 = \ell_1 + \ell_2, \tag{2.7}$$

where $k = K/L$ is the overall capital-labor ratio, $k_i = K_i/L_i$ is the sectoral capital-labor ratio, and $\ell_i = L_i/L$ is the share of sector i in the total labor force, so that $1 \geq \ell_i \geq 0$.

Using the assumption of CRS, the sectoral marginal and average productivities are uniquely determined by the sectoral capital-labor ratios. Label the average sectoral labor productivity, $Y_i/L_i = f_i(k_i)$, the first derivative of f, $f'(k)$, and per capita sectoral output, $Y_i/L = y_i$. Then per capita sectoral output is

$$y_i = \ell_i f_i(k_i), \tag{2.8}$$

and the competitive conditions are

$$r = p_i f_i'(k_i) \tag{2.9}$$

$$w = p_i[f_i(k_i) - k_i f_i'(k_i)] \tag{2.10}$$

We solve the system in terms of the wage-rental ratio. Under the competitive assumptions, the wage-rental ratio is equal to the ratio of marginal productivities of labor and capital, labeled ω. The basic relations are shown in Figure 2.1 where, ignoring sectoral notations, the average labor productivity is drawn as a function of the capital-labor ratio. For the value of k given by OD, the value of $f(k)$ is DA, $f'(k) = \tan \alpha$, $F_L = f(k) - kf'(k) = OB$, and using the triangle similarities, $\omega = OC$. It is now simple to see that ω is monotonically increasing in k. This monotonicity can also be established analytically:

$$\omega(k) = \frac{f(k)}{f'(k)} - k \quad \text{and} \quad \omega'(k) = -\frac{f(k) f''(k)}{(f'(k))^2} > 0. \tag{2.11}$$

By definition, ω is well defined for all k, and everywhere, $\omega'(k) > 0$.

The competitive condition implies the same ω for all sectors. The restriction of admissible allocation imposes restrictions on the value of ω. Some allocations that maintain the competitive conditions may require more resources than are actually available.

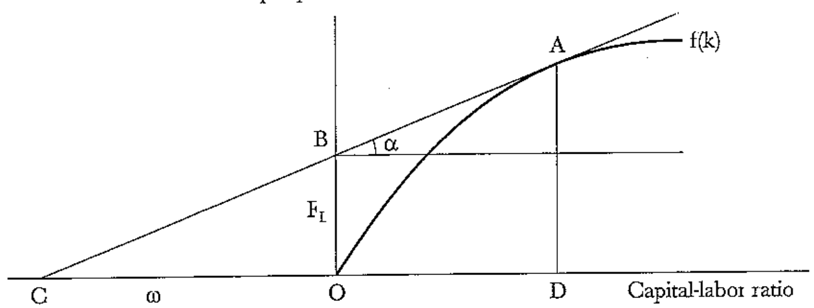

Figure 2.1 The production function

DEFINITION Wage-rental ratios that are consistent with admissible allocations are referred to as *admissible*

To characterize an admissible allocation we note that $\omega_i(k_i)$ is strictly monotonic so the inverse function, $k_i(\omega_i)$, exists. Let $\underline{k}(\omega) = \min[k_1(\omega), k_2(\omega)]$ and $\bar{k}(\omega) = \max[k_1(\omega), k_2(\omega)]$ Then ω is admissible if

$$\underline{k}(\omega) \le k \le \bar{k}(\omega). \tag{2.12}$$

The determination of the set of admissible wage-rental ratios is illustrated graphically in Figure 2.2. The intersection of the two functions $\omega_i(k_i)$ with the vertical line at k determines the boundaries of the admissible values of ω in the economy: $\underline{\omega}(k) = \min[\omega_1(k), \omega_2(k)]$ and $\bar{\omega}(k) = \max[\omega_1(k), \omega_2(k)]$. In what follows we deal only with $\omega \in [\underline{\omega}(k), \bar{\omega}(k)]$. By construction, an allocation is admissible if it is determined by an admissible ω.

The properties to be derived are related to the sectoral factor intensities Without a loss in generality sector 2 is assumed to be capital intensive

DEFINITION (factor intensity) Sector 2 is capital intensive if, for any admissible ω, $k_2(\omega) > k_1(\omega)$

Figure 2.2 is drawn in accordance with this assumption. Another definition is given below, and equivalent definitions are derived in the exercises Each of these definitions is useful in certain contexts

The solution of the supply side in terms of ω is now easily determined Let ℓ be the fraction of the labor force in the labor-intensive sector, sector 1 in our

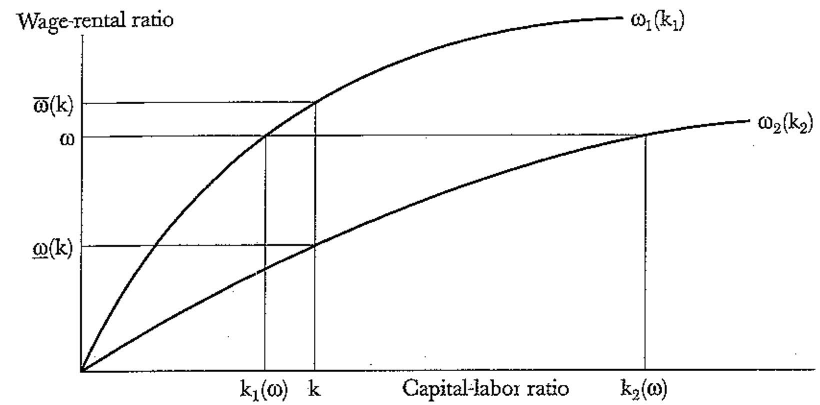

Figure 2.2 Factor ratio and factor-price ratio

case, and use equations (2.6) and (2.7) to obtain ℓ as a function of ω and k:

$$\ell(\omega, k) = \frac{k_2(\omega) - k}{k_2(\omega) - k_1(\omega)} \tag{2.13}$$

Combine (2.13) and (2.8):

$$\begin{aligned} y_1(\omega, k) &= \frac{k_2(\omega) - k}{k_2(\omega) - k_1(\omega)} f_1[k_1(\omega)], \\ y_2(\omega, k) &= \frac{k - k_1(\omega)}{k_2(\omega) - k_1(\omega)} f_2[k_2(\omega)]. \end{aligned} \tag{2.14}$$

The dependence of ℓ and y_i on k is indicated here for later reference In the present discussion k is assumed to be constant, and therefore ω is the only pertinent determinant. Having determined $k_i(\omega)$, the marginal productivities are determined. Using the competitive conditions, real factor prices are determined by conditions (2.9) and (2.10), and the product price ratio is given by

$$p(\omega) = \frac{f_2[k_2(\omega)]}{f_1[k_1(\omega)]} \tag{2.15}$$

Note that p, r, and w are functions of ω only and are independent of k. We refer to $p(\omega)$ as the price function. As shown in equation (2.16) below, $p(\omega)$ is monotone in ω. Thus the system can be solved in terms of any one of the endogenous variables and, in particular, in terms of p using equation (2.15).

The Response to Price Variations

The dependence of the system on prices is monotonic. To see this, use equation (2.13) to write

$$\frac{\ell(\omega, k)}{1-\ell(\omega, k)} = \frac{k_2(\omega) - k}{k - k_1(\omega)}$$

Because $k_i(\omega)$ is monotonically increasing in ω, the numerator increases and the denominator shrinks as ω increases. Hence, for $\omega^* > \omega$, $\ell(\omega^*, k) \geq \ell(\omega, k)$. This result follows from the full employment conditions in (2.13): $k_i(\omega)$ can increase with ω if and only if the labor-intensive sector expands and the capital-intensive sector shrinks. The simultaneous increase of $k_i(\omega)$ and $\ell(\omega, k)$ with ω implies a shift of resources to the labor-intensive sector. That is, L_1 and K_1 increase and L_2 and K_2 decline with ω. Consequently, Y_1 increases and Y_2 declines with ω. The discussion is now summarized:

PROPERTY 2.1 (resource allocation) An increase in the wage-rental ratio is associated with an increase in the employment of labor and capital by the labor-intensive sector and a corresponding decline in resource employment by the capital-intensive sector. Consequently, the output of the labor-intensive sector increases and that of the capital-intensive sector declines.

Since $k_i'(\omega) > 0$, we also have

PROPERTY 2.2 (factor prices) An increase in the wage-rental ratio is associated with an increase in the wage rate (w) and a decline in the rental rate (r).

Solving the system for all the admissible values of ω generates all the attainable combinations of y_1 and y_2 which, when plotted, generate the transformation curve. The transformation curve is the set of admissible production plans which are efficient in the sense that it is impossible to reallocate resources to increase the output of one product without decreasing the output of the other product. The concavity of the transformation curve follows from the concavity of the production functions (see the appendix to this chapter for a proof).

There is a monotone relationship between the product price ratio p and the factor price ratio ω. By the strong concavity of the transformation curve, the derivative dy_2/dy_1 increases in absolute value as y_1 increases. This derivative represents the marginal real cost of producing y_1 and given the competitive conditions, it is equal to p. But as $y_1(\omega)$ is monotone in ω, it is also monotone in p. This relationship between the product price ratio and the factor price ratio is the essence of the factor-price equalization theorem (Samuelson, 1948, 1949).

PROPERTY 2.3 (product-factor prices) The wage-rental ratio is monotonically increasing with the price of the labor-intensive product relative to that of the capital-intensive product.

Factor Intensity and the Price Equation

At this point it is appropriate to indicate that there are two distinct measures of factor intensity. The two measures are equivalent under the competitive conditions but, as shown in Chapter 8, when the competitive conditions are violated the two measures may diverge. The measure introduced now is related to the cost function, defined below, and it is expressed in terms of the factor shares. The share of capital in total output of sector i is $S_{iK} = rK_i/p_iY_i$ and that of labor is $1 - S_{iK} = S_{iL}$. Hence, $k_i(\omega)/\omega = rK_i/wL_i = S_{iK}/1 - S_{iK}$ and $k_2(\omega) > k_1(\omega)$ if and only if $S_{2K}(\omega) > S_{1K}(\omega)$. We introduce this result as a definition:

DEFINITION (factor-cost intensity) Sector 1 is *labor-cost intensive* if for any admissible ω, $I(\omega) \equiv S_{1L}(\omega) - S_{2L}(\omega) > 0$.

The price function depends on the cost intensity definition, as can be seen by deriving $p(\omega)$ using the cost function and the competitive condition. The cost function gives the lowest cost for producing output y for given factor prices:

$$C_i(w, r, Y_i) = \min_{K_i, L_i}[wL_i + rK_i; Y_i = F_i(K_i, L_i)]$$

As such, it is a function of the factor prices and output. The cost function takes on a simpler form when the production function is homogeneous of degree $\mu > 0$:

$$C_i(w, r, Y_i) = Y_i^{1/\mu}c_i(w, r).$$

Hence under CRS, $\mu = 1$, and $c_i(w, r)$ is the average cost. In competition, free entry and exit forces profits to zero, and therefore $c_i = p_i$. By definition, the cost function is homogeneous of degree 1 in w and r, which allows us to write $p_i = r\phi_i(\omega)$. Since $p = p_1/p_2$, we have the price function: $p = p(\omega)$. It follows from the envelope theorem (or Shephard's lemma) that $\partial c_i/\partial w = L_i(w, r, Y_i)/Y_i$ or $\partial \ln c_i/\partial \ln w = S_{iL}$ and $\partial \ln c_i/\partial \ln r = S_{iK}$. Write the total differential of p_i in its logarithmic form, using $\hat{x} \equiv dx/x$, to obtain

$$\hat{p}_i(w, r) = S_{iL}\hat{w} + (1 - S_{iL})\hat{r} = \hat{r} + S_{iL}\hat{\omega}$$

Hence

$$\hat{p}(\omega) = \hat{p}_1 - \hat{p}_2 = (S_{1L} - S_{2L})\hat{\omega} \equiv I(\omega)\hat{\omega} \tag{2.16}$$

Consequently, sign $\hat{p}(\omega)/\hat{\omega} = \text{sign } I(\omega)$. $I(\omega)$ is positive when sector 1 is labor-cost intensive; thus in this case $p(\omega)$ is monotone increasing in ω

This presentation has an important economic insight. When the wage rate increases proportionately more than the rental rate, the cost of production will increase more in the sector that pays a larger share of its product to labor. Note that the admissible values of p are bounded by $\underline{p} = \min[p(\underline{\omega}(k)), p(\overline{\omega}(k))]$ and $\overline{p} = \max[p(\underline{\omega}(k)), p(\overline{\omega}(k))]$.

When trading countries use the same technologies, they will have identical price functions. Trade between them leads to equality of p and therefore to equality of ω. But since the technologies are the same, factor prices will be the same. This is the meaning of the factor-price equalization theorem.

Since $-1 < I(\omega) < 1$, we have $|\hat{p}| < |\hat{\omega}|$; an increase in p will affect ω more than proportionately. Jones (1965) refers to this as the magnification effect and uses it to place boundaries on price changes.

Graphical Presentation of the Supply Side

The supply structure is summarized in Figure 2.3, which consists of four panels. The resource allocation is illustrated in panel I. Starting with an arbitrary wage-rental ratio ω_A, $k_{iA} \equiv k_i(\omega_A)$ are determined, and given a capital-labor ratio k, the resource allocation is determined, resulting in a production plan A marked on the transformation curve in panel IV. The supply price is determined by the slope of the transformation curve. It can also be determined directly by utilizing the price function presented in panel II. Thus, given ω_A, the price p_A is read directly from $p(\omega)$. Finally, panel III presents the supply function for sector 1, $y_1(p)$. It is obtained from $y_1(p) = y_1(\omega(p))$. As sector 1 is the labor-intensive sector and p is its relative price, the slope of the supply function is positive, $y_1'(p) \geq 0$. Knowing p_A, y_{1A} is determined. This corresponds to point A on the transformation curve.

It is now clear that the system can be solved in terms of variables other than ω. For instance, knowing y_1 or p makes it possible to solve for all the remaining variables. Similarly, the system can be solved in terms of per capita income, $y = py_1 + y_2$, measured in terms of nonagriculture. In Figure 2.3, given income level y_A, a tangent line is drawn from y_A to the transformation curve and thus p_A, y_{1A}, and y_{2A} are derived. If both y and p are given, it is possible that a tangent line does not exist, implying that the chosen y, given p, is inadmissible. In this case the economy specializes in one of the two sectors, depending on p.

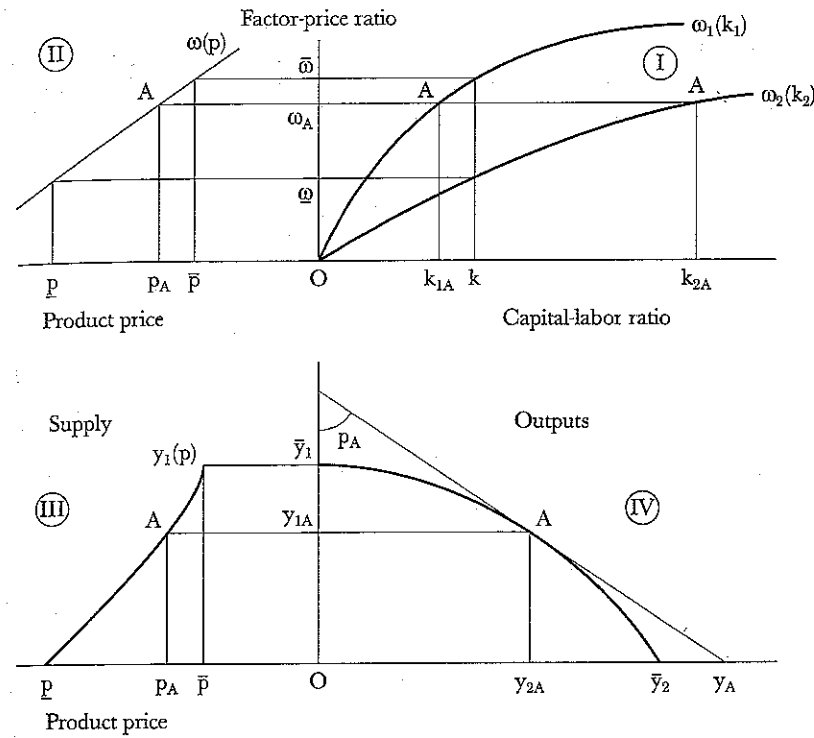

Figure 2.3 The supply side

This is a strong result which says that knowing y and p allows us to determine the output composition. However, the result has to be qualified by emphasizing that it is obtained conditional on technology and resources. Thus, its empirical validity is limited only to situations where these conditions are met.

Small Open Economy

The solution of the system in terms of p is of particular interest in that it describes the situation of a small open economy that trades in both products. In the absence of interventions, the country takes the international prices as given It is useful to repeat the foregoing results for such an economy by summarizing the response of the economy to an increase in the relative price of the labor-intensive product Such an increase in p leads to an increase in the wage-rental ratio, in the sectoral capital-labor ratios, and in the real wage, to a decline in the real rental rate, to a shift of both resources away from the capital-intensive to the labor-intensive sector, and therefore to an increase in the output of the labor-intensive sector and a decline in the output of the capital-intensive sector.

Alternatively, the consequences of a change in price can be evaluated by starting with the supply response. An increase in p leads to an increase in y_1 and a decline in y_2. Because p is the price of the labor-cost-intensive product, its rise causes a rise in ω and hence an increase in $k_i(\omega)$ in both sectors. All this can be inferred from Figure 2.3 We can now summarize the supply conditions

Summary of Supply

Admissible prices:

$$\omega \in [\underline{\omega}(k), \overline{\omega}(k)], \underline{\omega}'(k) > 0, \ \overline{\omega}'(k) > 0$$

Let p be the relative price of the labor-cost-intensive sector then,

$$p(\omega) > 0; p'(\omega) > 0$$
$$p \in [\underline{p}(k), \overline{p}(k)], \underline{p}'(k) > 0, \overline{p}'(k) > 0$$

Resource allocation:

$$k_i(\omega) > 0; k_i'(\omega) > 0, \qquad i = 1, 2.$$

Let ℓ be the proportion of the labor-intensive sector (sector 1) in total employment then,

$$1 \geq \ell(\omega, k) \geq 0; \quad \partial \ell(\omega, k)/\partial \omega > 0$$ [2]

Outputs:

$$\frac{\partial y_i(\omega, k)}{\partial \omega} \begin{matrix} > 0 & \text{for the labor-intensive sector} \\ < 0 & \text{for the capital-intensive sector} \end{matrix}$$

but always:

$$\partial y_1(p, k)/\partial p > 0$$
$$\partial y_2(p, k)/\partial p < 0$$

Demand

It is assumed that the preferences of consumers in the economy can be summarized by a well-behaved utility function. Under such an assumption, the equilibrium position in a closed economy is determined by a tangency of the indifference and the transformation curves. Such a solution is shown by point A in Figure 2.4 where $u(A)$ is the indifference curve with the utility level associated with the product bundle at A. Changes in supply conditions which lead to changes in the transformation curve will result in a new equilibrium point, to be determined in a way similar to the determination of A This result is too general, and as such it is not sufficiently informative Specifically, it does not use our empirical knowledge about the demand for agricultural output. Considerable insight can be gained by conducting the analysis in terms of demand The assumptions of a well-behaved utility function and rational choice yield demand functions which are homogeneous of degree 0 in nominal income and prices Using this property, we can write the demand in real terms, normalizing by one of the prices, say p_2:

$$x_1 = D_1(\underset{-\ +}{p, y}), \quad x_2 = D_2(\underset{+\ +}{p, y}), \tag{2.17}$$

where x_i is the per capita demand of the ith product and the underlying signs are those of the partial derivatives. We assume that the two products are normal (positive income coefficients), desirable and unsatiated so that the demand functions are asymptotic to the price-quantity axes. The assumption

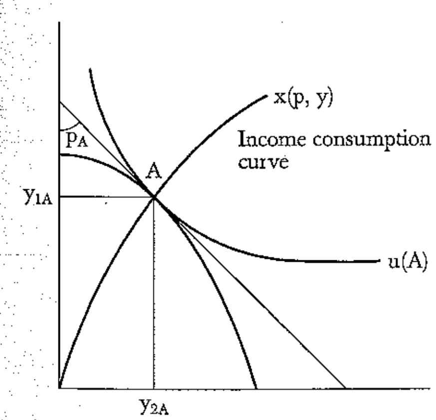

Figure 2.4 Demand

is justifiable when the commodities are broadly defined and it leads to[3]

$$\lim_{p\to\infty} D_1(p, y) = 0; \quad \lim_{p\to 0} D_1(p, y) = \infty$$

$$\lim_{p\to\infty} D_2(p, y) = \infty; \quad \lim_{p\to 0} D_2(p, y) = 0$$

In the graphical illustration we repeatedly use the income-expansion curve, which is the locus of $x_i = D_i(p, y)$ with p held constant [4] It is labeled $x(p, y)$ in Figure 2.4 Since the two products are assumed to be normal, it is possible to combine the two equations in (2.17) to

$$x_1 = D(\underset{-}{p}, \underset{+}{x_2}); \quad D(p, 0) = 0$$
$$\lim_{p\to\infty} D(p, x_2) = 0; \ \lim_{p\to 0} D(p, x_2) = \infty \qquad (2.18)$$

Wherever convenient, the compact notation $x_1(p, x_2) = D(p, x_2)$ is used

We make frequent use of the demand price, p^d, which is the price that corresponds to a given consumption bundle. Using the monotonicity of x_1 in p (see equation 2.18), the function can be inverted to yield

$$p^d = p(\underset{-}{x_1}, \underset{+}{x_2}). \qquad (2.19)$$

Equilibrium

The structure of the model meets the conditions of a well-behaved competitive economy, and consequently general results imply the existence of a competitive equilibrium. It is instructive to take advantage of the simple structure of the present model and to show graphically the determination of equilibrium. We will use this construction repeatedly in discussing the response of the system to various changes

In a closed economy, equilibrium is characterized by a price p that equates supply and demand For the small open economy the price is given, and trade is used to close the difference between the domestic supply and demand. Consequently, the analysis of equilibrium determination depends on whether the economy is open or closed. Although most economies trade in agricultural products and are largely price takers, we are still concerned with the analysis of a closed economy for two reasons. First, every economy has a nontradable sector which cannot be ignored in discussing the relationship between agriculture and nonagriculture In a related way, not all of the agricultural product is traded. This subject is introduced in Chapter 3, and its implications will be examined in the subsequent discussion Second, many of the agricultural

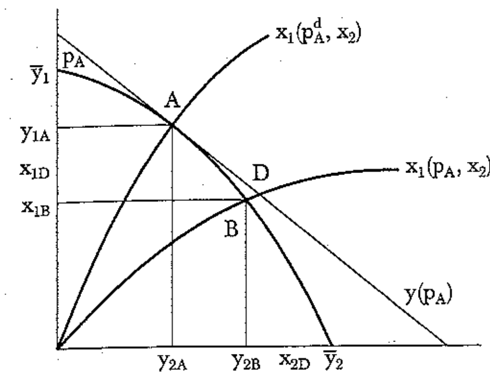

Figure 2.5 Equilibrium in a small open economy

problems are global and should be addressed as such. For that purpose, the closed economy framework is the relevant one.

Open Economy

The equilibrium characterization of a small open economy is as follows. Given the world price p, outputs $y_i(p)$ are determined, which in turn determine income $y(p)$. At this point, there is no domestic saving, so (per capita) consumption, $c(p)$, is equal to income plus net import, defined as the excess of import over export or trade deficit. To simplify, assume balanced trade so that $c(p) = y(p)$. The consumption of the ith product is determined by the demand function, $x_i[p, y(p)]$, or simply $x_i(p)$, and the sectoral identities are

$$y_i = x_i^c + x_i^e,$$

where $x_i^c = x_i(p)$ is per capita consumption and x_i^e is per capita net export. Since $y(p) = c(p)$, we have $px_1^e + x_2^e = 0$.

The solution is illustrated in Figure 2.5, where the world price is p_A The optimal production is given by A, and the corresponding income is $y(p_A)$ The demand curve corresponding to this price, $x_1(p_A, x_2)$, intersects the income line at D, yielding the optimal consumption. Because $y_{1A} > x_{1D}$, the country is an exporter of the agricultural product and an importer of the nonagricultural product, with export $y_{1A} - x_{1D}$ and import $x_{2D} - y_{2A}$.

Closed Economy

We begin the analysis with an arbitrary supply price, $p_A^s = p_A$, in Figure 2 5. The corresponding supply is at point A, and the demand is at B, not at D as for the open economy because now the consumption possibilities are bounded by the transformation curve and not by the market line. Thus at p_A there is an excess supply of agriculture, $x_1(p_A) - y_1(p_A) < 0$, implying that p_A is too high for the market to clear. A decline in the price will move the production point to the right and the consumption point to the left. Eventually, a market-clearing price is achieved.

More formally, to search for an equilibrium price, examine the demand price that will induce the consumption of the output at A. This is obtained by using equation (2 19): $p_A^d = p(x_{1A}, x_{2A} \mid x_{iA} = y_{iA})$ Because $y_{1A} > x_1(p_A)$, we have $p_A^d < p_A$. Can instead p_A^d be an equilibrium price? The answer is no because as p declines from p_A to p_A^d, the output $y_1(p)$ declines, and the optimal output at p_A^d is below and to the right of A, resulting in an excess demand for agriculture, $x_1(p_A^d) - y_1(p_A^d) > 0$. Therefore p_A^d is too low for the market to clear. We have thus established boundaries on the equilibrium price and by the continuity of the demand and supply functions, there exists an equilibrium price p_E, such that $p_A^d < p_E < p_A^s$, and $x_1(p_E) = y_1(p_E)$. By construction, this price also clears the nonagricultural market.

The Restricted Demand

The analysis can also be illustrated in terms of the supply and demand equations familiar from partial equilibrium analysis In so doing, we should keep in mind that in our framework the supply and demand are obtained subject to the economywide constraints and as such are not partial equilibrium functions. To find the equilibrium price we have to search for p such that $x_i(p) = y_i(p)$. To do so, we replace x_2 in (2.18) with the value of y_2 on the transformation curve that corresponds to the price p, $x_2 = y_2(p)$, to obtain

$$x_1 = D[p, x_2 \mid x_2 = y_2(p)] = x_1(p) \qquad (2\ 20)$$

We refer to (2 20) as the restricted demand

DEFINITION (restricted demand) The *restricted demand* is obtained by restricting the demand $x_1(p, x_2)$ to the domain $x_2 = y_2(p)$.

The concept is illustrated in Figure 2 6. Starting with $\underline{p}$, draw $x_1(\underline{p}, x_2)$ in the right panel and continue it to point A where $x_2 = y_2(\underline{p}) \equiv \overline{y}_2$ Thus point A meets the condition of the restricted demand, and it is marked in the left

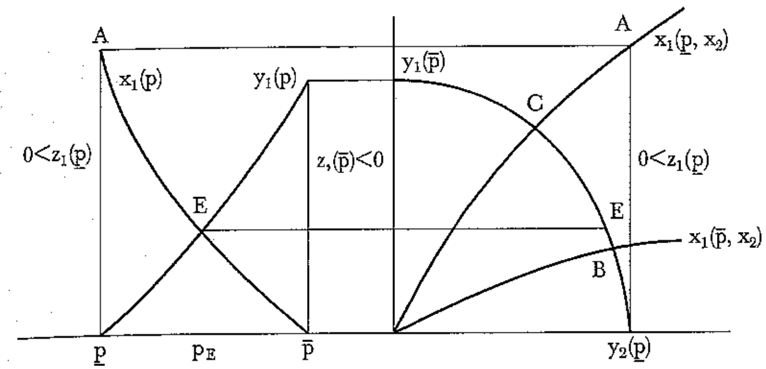

Figure 2.6 Restricted demand: Closed economy

panel as a point on the function $x_1(p)$. Turning to the other extreme, at $\bar{p}$, $y_2(\bar{p}) = 0$, and hence $x_1(\bar{p}, 0) = 0$. This procedure can be repeated for other prices. The restricted demand is continuous, and it is drawn in the left panel joining the points illustrated above. The supply $y_1(p)$ is obtained as before, and the intersection of supply and demand gives the equilibrium point E

The difference between the restricted demand and the supply is the excess demand:

$$z_1(p) = x_1(p) - y_1(p)$$

Note that $z_1(p)$ is everywhere declining since $x_1'(p) < 0$ and $y_1'(p) > 0$. The graphical derivation of the restricted demand also generates the excess demand In terms of the left-hand panel, it is the vertical distance between $x_1(p)$ and $y_1(p)$. Thus, at $\underline{p}$, the excess demand is simply $x_1(\underline{p})$, since $y_1(\underline{p}) = 0$. Similarly, at $\bar{p}$, $x_1(\bar{p}) = 0$. The reason is that at this point there is no production of y_2 and therefore no consumption of this product. By the assumptions on demand, zero consumption of a product can only take place when there is no consumption at all. In this case, $z_1(\bar{p}) = -\overline{y}_1$.

The analysis shows that for low p, $z_1(p) > 0$, for high p, $z_1(p) < 0$, and there is a price, p_E, such that $z_1(p_E) = 0$ Convergence to equilibrium is achieved when a positive excess demand generates an increase in p:

$$z_1(p)dp \geq 0$$

The equality is achieved when both $z_1(p)$ and dp are equal to zero

The same analysis can also be carried out for an open economy. However, in this case, the values of the excess demand are different in that for any p the

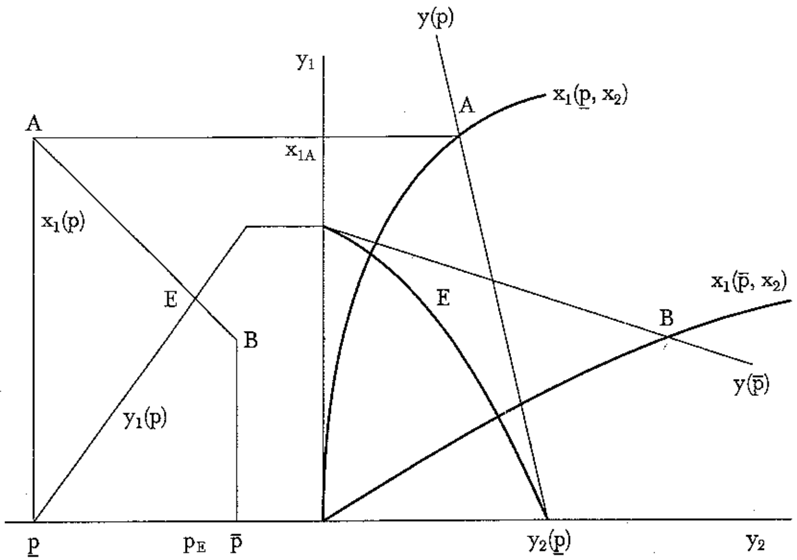

Figure 2.7 Restricted demand: Open economy

consumption possibilities are represented by the market line rather than the transformation curve It is therefore more meaningful for the presentation of an open economy to restrict the consumption of x_2 to values of $y_2(p)$ that are attainable at value output $y(p) : x_1(p) = D[p, x_2(p, y(p))]$, where $y(p)$ is the income corresponding to price p.

The derivation of the restricted demand for an open economy is portrayed in Figure 2.7. When the world price is $\underline{p}$, the economy specializes in sector 2. The generated income, $y(\underline{p})$, represents the budget constraint It intersects the income expansion curve $x_1(\underline{p}, x_2)$ at A in the right panel Point A marked on the restricted demand in the left panel has coordinates $\underline{p}$ and $x_{1A} = x_1(\underline{p})$. Similarly, point B corresponds to $\overline{p}$ At E supply and demand intersect, and there is no trade Therefore E is also a point on the restricted demand of a closed economy (not shown in the figure).

Equal Factor Intensities

The foregoing discussion does not properly cover the case of equal factor intensities, where $k_1(\omega) = k_2(\omega)$ Such equality may hold locally where $k_i(\omega) = k$,

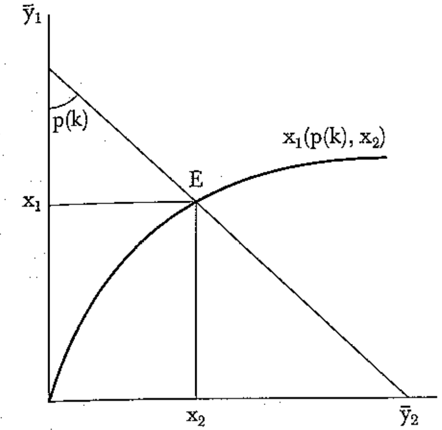

Figure 2.8 Equilibrium: Equal factor intensities

or in the large, in which case the $k_i(\omega)$ functions in the two sectors are identical In either case equation (2.6) cannot be used to determine ℓ, and the subsequent discussion based on this equation has to be modified. Still, the economy does have an equilibrium point, and its determination is rather simple.

Under equal factor intensities the transformation curve is a straight line Since by assumption $k_1(\omega) = k_2(\omega) = k$, we have $\omega = \underline{\omega}(k) = \overline{\omega}(k)$ That is, the admissible value of ω for the economy is unique, and it is a function of k, $\omega(k)$. Since p is uniquely determined by $p(\omega)$, it has a constant value determined by $p(\omega(k)) = p(k)$

Utilizing (2.8), the intercepts of the transformation curve are $\overline{y}_i = f_i(k)$. Utilizing the competitive conditions, $p = f_2'(k)/f_1'(k)$, and since k is constant, so is p In other words, the transformation curve represents the combination of outputs $\ell f_1(k)$ and $(1 - \ell) f_2(k)$. Consequently,

$$dy_2/dy_1 = \frac{f_2(k)}{f_1(k)} \frac{d(1-\ell)}{d\ell} = -\frac{f_2(k)}{f_1(k)} = -\frac{\overline{y}_2}{\overline{y}_1}$$

All this is shown in Figure 2.8, where we read $p(k)$ directly from the transformation curve. The equilibrium, E, is determined graphically by drawing the demand curve corresponding to $p(k)$, resulting in consumption (x_1, x_2).

Income Distribution

Differences in income among individuals arise due to differences in the attributes that generate and affect income. Growth changes the supply and demand of these attributes, their market values, and therefore the income distribution. The most basic attributes are the ownership of factors of production, and the starting point for any discussion is to trace the effect of growth on the returns to factors of production. At this stage of static analysis some preliminary concepts are presented that are related to the more general discussion below.

Income generated by production is distributed as wages to labor and rental to the owners of the capital good. Variations in factor prices affect the income distribution between labor and capital. When the demand depends only on the level of income and not on its source, the income distribution is determined by the equilibrium position but does not affect it. A change in factor prices may increase or decrease the share of labor in total income (labor share) depending on the ease of factor substitution to be measured by the elasticity of substitution (ES).

For the case of a two-factor production function, the ES is defined by

$$\sigma = d \ln k / d \ln \omega \tag{2.21}$$

The functional distribution of income can be measured by the ratio of the labor share to that of capital:

$$\theta_i \equiv S_{iL}/(1 - S_{iL}) = wL_i/rK_i = \omega/k_i.$$

Thus, sign $\partial S_{iL}/\partial\omega =$ sign $\partial\theta_i/\partial\omega = \text{sign}(1 - \sigma_i)$. Consequently, the share of labor in the total income of sector i increases (decreases) with the wage-rental ratio when $\sigma_i < 1 (\sigma_i > 1)$. It is unaffected by the wage-rental ratio when $\sigma_i = 1$. If everywhere $\sigma_i = 1$, the production function is Cobb-Douglas.

The relationship between σ_i and the concavity of $\omega_i(k_i)$ is illustrated in Figure 2.9.[5] As ω increases, θ decreases for sector 1 ($\sigma_1 > 1$) and increases for sector 2 ($\sigma_2 < 1$). An elasticity of substitution larger than 1 describes a situation where an increase in ω causes more than a proportional substitution of capital for labor.

It is clear from the foregoing discussion that a given change in ω may affect the labor shares in the two sectors in different directions when $(\sigma_1 - 1)(\sigma_2 - 1) < 0$. In this case, the net effect of such a change on the labor share in the economy at large depends on the relative importance of the sectors and the change in the sectoral composition called for by the change in ω.

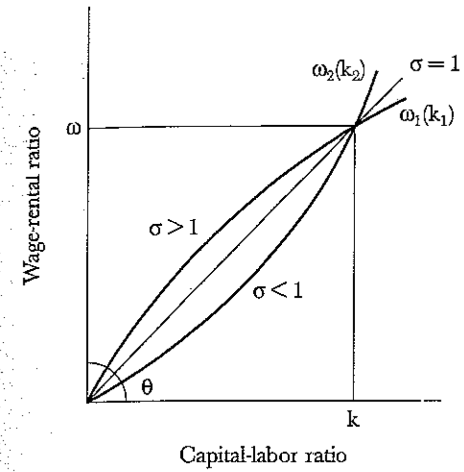

Figure 2.9 Factor intensities and factor price-ratio

Much of the discussion on agricultural policy deals with a comparison of income in agriculture with that in nonagriculture. This is related to another dimension of income distribution. Let $S_1 = py_1/y$ be the proportion of agriculture in total value output. In terms of Figure 2.3, it is the ratio $(y_A - y_{2A})/y_A$. Clearly, this ratio increases monotonically with y_1. Thus under the conditions of the model, changes leading to an increase in y_1 increase the share of agriculture in total output.

The Introduction of Land

Agricultural production requires land; therefore it must be introduced into the analysis. As a first step, we turn now to show how the foregoing results should be interpreted when land is included under the assumption that land is of homogeneous quality. This assumption is removed in a more detailed discussion of land in Chapter 4 and thereafter.

We write the agricultural production function with land (A) included and with population held constant, and therefore ignored, as

$$Y_1 = A^{1-\mu}[F_1(K_1, L_1)]^{\mu}, \tag{2.22}$$

where F_1 is concave, twice differentiable, linear homogeneous, and $0 < \mu < 1$. The degree of the function, μ, can be a function of various variables, including land However, locally it is considered to be constant Consequently, by a simple transformation we can write

$$Y_1^* = (Y_1)^{1/\mu} = A^\epsilon F_1(K_1, L_1),$$

where $\epsilon = (1-\mu)/\mu$ Let $y_1^* = Y_1^*/L$, and write

$$y_1^* = A^\epsilon \ell f_1(k_1). \tag{2.23}$$

Thus instead of dealing with per capita agricultural output, y_1, we deal with a monotonic transformation of it, y_1^*. To derive the actual output from y_1^*, simply reverse the transformation: $y_1 = y_1^{*\mu}$

Under this specification, land is weakly separable from the other inputs in that the marginal rate of substitution of capital and labor is independent of the size of land. Consequently $\omega_1(k_1)$ remains unchanged, as does the solution for the resource allocation, ω and $\ell_i(\omega)$. Figure 2 10 illustrates the transformation curves between y_2 and y_1^* as well as y_1 Without loss in generality, we assume now that land is held constant.

The supply price of the transformed output, labeled p^*, reflects the fact that its production function is linear homogeneous. However, the production function of actual output displays decreasing returns to scale in capital and labor. Therefore the price of the actual output will be higher than that of the transformed output, except for small outputs In deriving the supply price it is assumed that there is no alternative use for land, and therefore the return to land is rent. Thus the cost function that corresponds to (is dual to) actual output is written as $C = y^{1/\mu}\phi(w, r)$, and the average cost is $c = y^\epsilon \phi(w, r)$. Applying this to sector 1, we have $p_1 = C_1/y_1 = y_1^\epsilon \phi(w, r)$. The transformed variable is linear homogeneous, and we can attach to it an average cost, $p_1^* = \phi(w, r)$ Thus

$$p = \frac{p_1}{p_2} = y_1^\epsilon \frac{\phi(w, r)}{p_2} = y_1^\epsilon \frac{p_1^*}{p_2} = y_1^\epsilon p^*. \tag{2.24}$$

The price functions $p^*(\omega)$ and $p(\omega, y_1)$ are drawn in panel II of Figure 2.10, and the supply functions $y_1(p)$ and $y_1^*(p)$ are drawn in panel III [6]

To obtain the equilibrium solution, demand must be expressed in the same units as supply The quantity demanded is then transformed by $x_1^* = x_1^{1/\mu}$, and as such it can be expressed as a function of p and x_2:

$$x_1^* = [D(p, x_2)]^{1/\mu}. \tag{2.25}$$

The foregoing analysis conducted by ignoring land applies immediately to the transformed variable y_1^*. To obtain the results in terms of the original

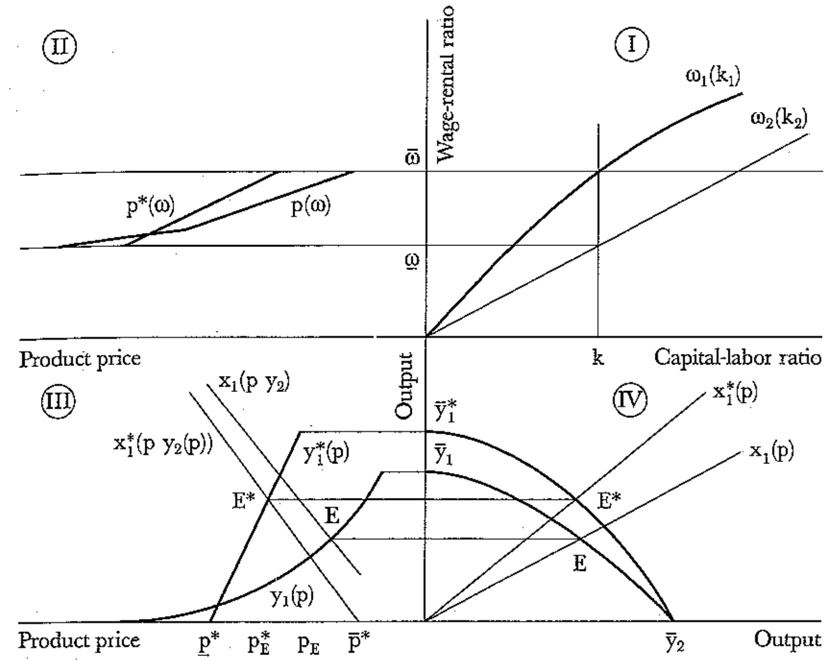

Figure 2.10 The economy with land

variables, we transform y_1^* and x_1^* to y_1 and x_1 respectively. The model is illustrated in Figure 2.10 in terms of the two systems. Qualitatively, the supply price of agricultural output increases more sharply with y_1 than with y_1^*. This is shown in panel III of Figure 2 10 as a downward shift of the supply function for y_1. In this sense, much of the qualitative analysis can be conducted by ignoring land. Land becomes important when dealing with specific issues related to land itself such as: (1) identifying the determinants of the returns to land, (2) land as a source of farm income, (3) the determinants of the size of the arable land, and (4) the effect of land scarcity on the composition of the implemented technology in agriculture These and related issues are addressed in subsequent discussion in later chapters

Under the present specification, using the Euler condition for a CRS production function, $1-\mu$ is the factor share of land. Denote the rent on land, measured in terms of y_2, by R, and the population by L, then

$$R = (1-\mu) p y_1 L/A. \tag{2.26}$$

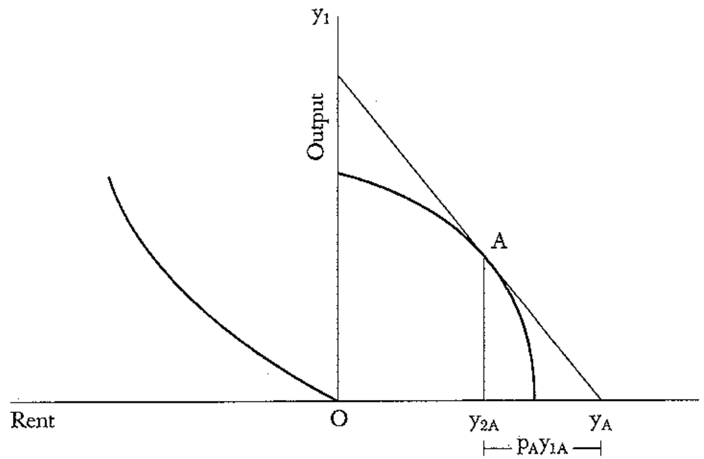

Figure 2.11 Rent and the level of activity

With A and $(1-\mu)$ held constant, the rent increases monotonically with the value of the agricultural output, py_1 But in view of the supply function, R is monotone increasing with either y_1 or p.

The rent can be shown graphically as in Figure 2 11 by drawing a tangent to point A with an intercept $y_A = p_A y_{1A} + y_{2A}$ and $p_A y_{1A}$ is given by the segment $y_A - y_{2A}$ Note that rent varies from 0, when the economy specializes in nonagriculture, to $(1-\mu)L\overline{p}\,\overline{y}_1/A$. Clearly, an increase in the ratio of population to land increases the rent

Food and Agriculture

In much of the public discussion and economic analysis, food is assumed to be identical with agriculture, and this is the approach taken here. However, for some purposes this assumption is too restrictive in that it does not allow for the fact that food contains other inputs originating in nonagriculture such as processing, packaging, and transportation, as well as food consumed in restaurants. The quantity of the nonagricultural inputs and their prices affect the consumer price of food. This raises two questions: first, what effect should it have on our analysis and second, what is the empirical implication This topic is discussed in Mundlak (1986), and what follows summarizes results pertinent to our discussion

Let the utility function of a representative consumer be written as

$$u = U[F(A, Q), N]. \qquad (2.27)$$

The function is weakly separable in food, F, and nonfood, N. Food has an agricultural component, A, and a nonagricultural component, Q. The ratio $q = Q/A$ can serve as a measure of quality of food [7] Since the price of Q and N is the same, we can view $Q + N = \overline{N}$ as a composite good, and write the utility function as $U(A, \overline{N})$, resulting in demand functions unambiguously signed:

$$A(p, u), \quad \overline{N}(p, u). \qquad (2.28)$$

To analyze the short-run equilibrium we utilize (2 28) to derive the demand function

$$x_1 = D(\underset{-}{p}, \underset{+}{x_2}), \qquad (2\ 29)$$

where x_1 is per capita demand of A, and x_2 is per capita demand of $\overline{N}$ The function has the properties assumed for (2 18) Consequently, it can be shown that a unique stable short-run equilibrium is established which determines p, A, and $\overline{N}$ (Mundlak, 1986).

Empirical Evidence

The focus here is on demand alone, because supply is discussed at length in Chapter 14

Demand

Demand plays a crucial role in the determination of the dynamics of agriculture. Particularly, sectoral growth depends on income elasticities The role of demand is related to Engel's law, which states that the proportion of the consumer's budget spent on food declines with income. In other words, the utility function is not homothetic, and the income elasticity for food is less than one. In fact, it is much lower than one. Empirical work has been conducted with aggregate time-series data, and with household data. In general, price elasticities can be obtained more reliably from time-series data, where there are more fluctuations in prices. However, such analysis is complicated by other factors, so it will not be reviewed here except to indicate that the price elasticities of demand for the aggregate product are low, well below one (in absolute value).

On the other hand, income elasticities can be obtained by analyzing micro data where there exists a big spread in income between individuals. In general, the message from such studies is quite robust Early empirical estimates of income elasticities are reviewed by Houthakker (1957), who concludes the analysis by stating that "[i]f no data on the expenditure patterns of a country are available at all, one would not be very far astray by putting the partial elasticity with respect to total expenditure at .6 for food, 1 2 for clothing, 8 for housing, and 1 6 for all other items combined and the partial elasticity with respect to family size at .3 for food, zero for housing and clothing, and $-.4$ for miscellaneous expenditures" (p 550). More recent results obtained with more advanced estimation methods further confirm the earlier results. As indicated above, the income elasticities for food expenditures are likely to exaggerate the income elasticities of demand for the agricultural product when the income elasticity for quality exceeds that for agriculture A positive relationship between food quality and income in consumer budget analysis was reported by Prais and Houthakker (1955, 1971) and by subsequent studies.

Above we interpreted the increased quality with a relative decline of the agricultural component in food. This is reflected in the cost composition of food at the retail level. The share of agriculture in the retail cost of food in the United States declined from a level of 50 percent in the mid-1940s to 33 percent in 1983 (Dunham, 1984). More specifically, over the period 1946–1982, this share declined at an average annual compounded rate of 0 35 percent. Over the same period, the terms of trade, p, declined by an average annual rate of 1.16 percent, whereas per capita income and the quality measure, as defined in the previous section, increased by 2 1 and 0.3 percent respectively. Additional empirical evidence for other countries is reviewed in Mundlak (1986).

Discussions of the demand of low-income individuals focus on the nutrient content of food, which does not reflect nonagricultural inputs. Strauss and Thomas (1998) examined the literature dealing with the extent to which wealth affects nutrition and the methodologies used in these studies. They report a positive caloric response to income up to a given level of income, and thereafter the response is weak. Orders of magnitude for income elasticities of caloric intake of low-income individuals are 0 35–0 55. The income elasticity for aggregate agricultural demand reflects the demand of all income groups and therefore is likely to be below these values, particularly in the more developed countries.

Factor Intensity and Agriculture

Because many of the results depend on factor intensity, it is important to examine whether agriculture is labor or capital intensive We now know that there are two different concepts of factor intensity, one related to factor ratios and one related to factor shares. The first one affects the response of factor

allocation to change in factor prices, and the second determines the relationship between factor and product prices. It is of interest to compare the two measures, but this is not a simple matter because of limitations on data on sectoral capital stock. We return to this in Chapter 10. At this point we look instead at some indirect measures that can be confronted with data more readily available Let $\rho = K_1/K$ be the share of agriculture in the capital stock, and note that

$$[k_1(\omega) - k_2(\omega)][\rho(\omega) - \ell(\omega)] \geq 0$$

The equality holds when the two terms are simultaneously equal to zero. This is, however, of little help when ρ is unknown. A more useful observation is that the share of agriculture in total output, $S_1 \equiv pY_1/Y$, can be expressed as a weighted average of the shares of agriculture in resources,

$$S_1 = \frac{wL_1 + rK_1 + RA}{Y} = S_L\ell + S_k\rho + S_A S_1,$$

where $S_L = wL/Y$, $S_K = rK/Y$, and $S_A = RA/pY_1$. We will concentrate on the relation between S_1 and ℓ. For this purpose we can aggregate physical capital and land and rewrite $S_1 = S_L\ell + (1 - S_L)\rho_T$, where ρ_T is the share of agriculture in capital, including land. If agriculture is labor intensive, we should observe $\ell > S_1$. The data on S_1 and ℓ are presented in Figures 1 14 and 1 15 and are summarized in Figure 2.12, which reports the distribution of the ratio S_1/ℓ for 68 countries in 1950 and 1990. On the whole, this ratio is well below one.

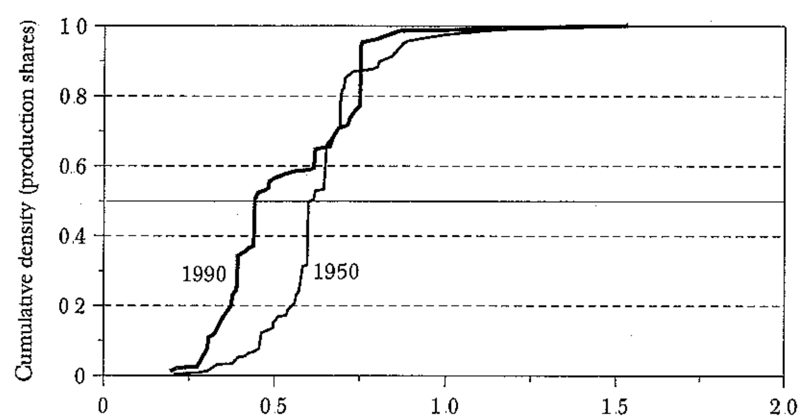

Figure 2.12 Agriculture: Ratio of share in GDP to share in labor force, 68 countries (Based on database of Mundlak, Larson, and Crego, 1998)

Turning to the measure of cost-factor intensity, we may compare the sectoral factor share of labor. Such a comparison is also restricted by data problems. In general, however, the labor share in agriculture is smaller than that in nonagriculture, indicating that agriculture is cost-capital intensive. Thus there seems to be a contradiction between the two measures of factor intensity. Two possible explanations come to mind. First, when we take land into consideration, a low value for the factor share of labor may be due to the factor share of land and not to a high value for the factor share of capital. In this case, there is no contradiction with the direct measure, but the implications seem nevertheless to indicate a contradiction. An increase in the wage rate will affect the cost of production in nonagriculture more than in agriculture. Second, it is possible that the assumption of equal factor prices across sectors, on which the equivalence of the two measures is based, is violated. This conclusion is consistent with the notion that the wage rate in agriculture is lower than in nonagriculture. We return to this subject in Chapters 8 and 9 (see also exercises 2.8–2.10)

Appendix 2A

1. Bibliographic note.

The basic structure of supply in a two-sector economy can be found in Stolper and Samuelson (1941) and Samuelson (1948, 1949). It was adopted for growth analysis by Uzawa (1961, 1963). A uniform exposition is given by Jones (1965), following a somewhat different route from that used here. Although Jones's main analysis is for the small open economy, he also allows for product substitution on the demand side. Growth considerations require allowance for income effect on demand. This is introduced in Mundlak (1965) and further extended by Mundlak and Mosenson (1967, 1970). An early empirical formulation of the relationship between agriculture and the rest of the economy appears in Tolley and Smidt (1964)

2. The set of all admissible allocations is convex.

Proof: Consider two admissible allocations:

$$\lambda' k_1' + (1-\lambda')k_2' \le k$$

$$\lambda'' k_1'' + (1-\lambda'')k_2'' \le k.$$

What we want to show is that any convex combination of admissible allocations is also admissible. To form a convex combination of such

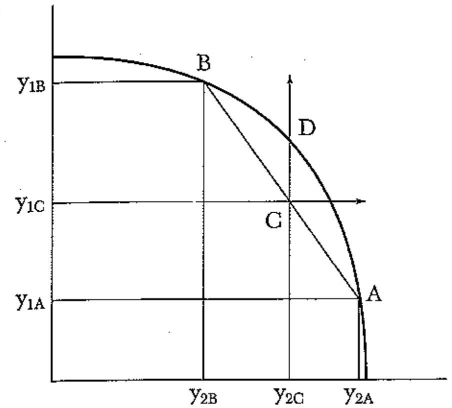

Figure 2A.1 Concavity of the transformation curve

admissible allocations, let $0 \le a \le 1$, multiply the first inequality by a and the second by $(1-a)$, and add them up:

$$a\lambda' k_1' + a(1-\lambda')k_2' + (1-a)\lambda'' k_1'' + (1-a)(1-\lambda'')k_2'' \le k.$$

This can be written in the form:

$$\tau \bar{k}_1 + (1-\tau)\bar{k}_2 \le k,$$

where $\tau = a\lambda' + (1-a)\lambda''$; $0 \le \tau \le 1$ and $\bar{k}_1 = [a\lambda' k_1' + (1-a)\lambda'' k_1'']/\tau$ and a similar expression for the other components. ■

3. The concavity of the transformation curve.

We show that the concavity of the transformation curve follows from the concavity of the production functions. Let ω_A and ω_B be two admissible values of ω, and y_{iA} and y_{iB} be the corresponding outputs. To simplify the discussion, normalize the labor force to $L = 1$. For any $0 \le \lambda \le 1$, define point C with outputs: $\bar{y}_i = \lambda y_{iA} + (1-\lambda)y_{iB}$. Points A, B, and C are shown in Figure 2A.1. To prove concavity it is necessary to show that there exists a feasible production plan, say D, such that $y_{iD} \ge \bar{y}_i$.

Consider the allocation $\bar{k}_i$, $i = 1, 2$: $\bar{k}_i = \lambda k_{iA} + (1-\lambda)k_{iB}$, which is admissible because it is a convex combination of admissible allocations. Let $\bar{y}_i \equiv \lambda \ell_{iA} f_i(k_{iA}) + (1-\lambda)\ell_{iB} f_i(k_{iB})$, label $\bar{\ell}_i = \lambda \ell_{iA} + (1-\lambda)\ell_{iB}$, and $y(\bar{k}_i) \equiv \bar{\ell}_i f_i(\bar{k}_i)$, then due to the concavity of the production functions, $y_i(\bar{k}_i) > \bar{y}_i$. Consequently, if $\bar{\ell}$ represents an optimal allocation, the proof is completed; otherwise, there exists an optimal allocation with a larger output Thus for any initial points A, B and for any $\lambda \in [0, 1]$ there exists an admissible allocation that dominates point C

4. The unit profit.

In some instances it is useful to deal with a somewhat different convergence rule which states that whenever the cost of production is below the demand price, output will increase. Define $\pi_1(y_1) = p^d(y_1) - p^s(y_1)$ as the unit profit in sector 1. Equilibrium is achieved by finding y_1 such that $\pi_1 = 0$. The demand price, defined in (2.19), is evaluated here only for quantities on the transformation curve:

$$p^d(\underset{-}{x_1}, \underset{+}{x_2} \mid x_i = y_i; \underset{-}{y_2(y_1)}) = \underset{-}{p^d(y_1)}.$$

In terms of Figure 2.6, the demand price at C is obtained from $p^d(y_{1C}, y_{2C}) = p^d(y_{1C}) = \underline{p}$ Similarly, at B, $p^d(y_{1B}) = \overline{p}$

The supply is drawn as before except that now it is expressed as $p^s(y_1)$ Equilibrium is established at E, where $p^d(y_{1E}) = p^s(y_{1E})$. For outputs $y_1 > y_{1E}$ the profit is negative. Thus, $\pi(y_1)$ alternates sign at E, and it is monotone declining in y_1:

$$\frac{d\pi(y_1)}{dy_1} = \frac{dp^d(y_1)}{dy_1} - \frac{dp^s(y_1)}{dy_1} < 0.$$

The convergence rule is $\pi_1(y_1)dy_1 \geq 0$, with equality achieved only when the two terms are identically zero. Note that the convergence rule is maintained if and only if the system is stable.

5 Results using duality.

The discussion in this volume is primarily related to resource allocation and technology choice. This has dictated the form of the analysis. Some results can be made more compact, however, by using duality theory This has already been demonstrated above in the use made of the cost function and will also be used in subsequent chapters. Some of the discussion is therefore repeated here, in particular the characterization of the equilibrium conditions using duality

Revenue functions In the discussion of the restricted demand in an open economy, a reference was made to $y(p)$, the value of output obtained as

a result of the optimization on the supply side. This is in fact the value of the revenue function defined as

$$R(p, k, T) = \max_y(py; (y, k) \in T).$$

This definition introduces some new notation. The term py is the inner product $p_1y_1 + p_2y_2$, and recall that in the present discussion $p_1 = p$ and $p_2 = 1$. The expression $(y, k) \in T$ implies that we only consider vectors $y = (y_1, y_2)$ that can be produced with inputs k and technology T. Thus $R(p, k, T)$ indicates the maximum revenue attainable with (p, k, T).

In the present discussion outputs are expressed on a per capita basis, and the resources are summarized by the capital-labor ratio k The definition of R is general in that k can be a vector of J inputs and y can be a vector of I outputs.

The revenue function is nondecreasing, convex, and linear homogeneous in p By Hotelling's lemma, the partial derivatives are equal to the supply functions $R_p(p, k, T) = y(p, k, T)$. When it is understood that the technology is constant, there is no need to mention T explicitly in the argument of R. Similarly, when resources are fixed ("short run"), there is no need to include k in the argument. This has been the case in the foregoing discussion of supply Nevertheless, at this point, where the properties of R are summarized, it is also useful to summarize its behavior in the inputs, and therefore k is mentioned here explicitly. Specifically, $R(p, k)$ behaves in k like a production function where R is a measure of output, known as a Hicks composite good. It is obtained by aggregating the outputs for a given price R is nondecreasing and concave in k. The partial derivatives of R with respect to k give the factor (shadow) price: $R_k(p, k) = w$, where w here is the vector of the prices of the elements of k In our particular case, k is the capital-labor ratio, and hence its price is $r/w = 1/\omega$ Note that we do not differentiate in the notation between vectors and scalars.

Demand Let $u(x)$ be a well-defined utility function of x. The uncompensated demand functions, written in vector notation, $x(p, y)$, are obtained from the solution to

$$\max_x[u(x); y \geq px].$$

Substituting $x(p, y)$ for x in $u(x)$ gives the indirect utility function, which indicates the maximum level of utility that can be attained at (p, y):

$$v(p, y) = u[x(p, y)].$$

The compensated demand function, $h(p, u)$, is obtained as a solution to

$$\min_x [px; u(x) \geq u]$$

Substituting $h(p, u)$ in the budget constraint gives the expenditure function, indicating the minimum cost that has to be paid for achieving u at price p:

$$e(p, u) = ph(p, u)$$

The expenditure function is nondecreasing in p, linear homogeneous in p, concave in p, continuous in p with all prices strictly positive and

$$e_p(p, u) = h(p, u)$$

Since $e(p, u)$ is linear homogeneous in p, $h(p, u)$ is homogeneous of degree 0 in p, implying

$$ph_p(p, u) = 0.$$

The equivalence of the maximum and minimum optimization setups produces the following identities:

$$x(p, y) = h[p, v(p, y)]$$
$$h(p, u) = x[p, e(p, u)].$$

Differentiation of the latter gives the Slutsky equation

$$\frac{\partial h_i}{\partial p_j} = \frac{\partial x_i}{\partial p_j} + x_j \frac{\partial x_i}{\partial y},$$

which can be written in terms of elasticities:

$$e_{ij} = E_{ij} + \pi_j E_{ic},$$

where

$$e_{ij} = \partial \ln h_i / \partial \ln p_j,\ E_{ij} = \partial \ln x_i / \partial \ln p_j,\ e_{iy} = \partial \ln x_i / \partial \ln y,$$

and $\pi_j = p_j x_j / y$.

Equilibrium We can now restate the equilibrium conditions. The equality of income and consumption can be expressed as the equality of the values of the revenue and expenditure functions.

$$R(p, k) = e(p, u).$$

This equality yields the solution u^o conditional on k and p. In the closed economy p is determined by

$$R_p(p, k) = e_p(p, u).$$

Given the solution, p^o and u^o, $e_p(p^o, u^o)$ and $R_p(p^o, k)$ indicate the equilibrium values of consumption (x_i^o) and production (y_i^o) respectively. By construction, $y_i^o = x_i^o$. In the small open economy, p^o is given, and $R_p(p^o, k) - e_p(p, u^o)$ is the vector of net exports.

Exercises

2.1 Draw the four-panel production diagram for the case where agriculture (sector 1) is capital intensive. Mark the limiting values for ω, p, and y_i wherever applicable. Summarize in words the qualitative changes that occur in each of the panels due to the change in factor intensity of agriculture from labor to capital intensive

2.2 Draw a modified panel III where the vertical axis is the ratio y_1/y_2. Do it for the two possibilities of factor intensities.

2.3 Draw another modified panel III where now the horizontal axis is the wage-rental ratio (ω). Do it for the two possibilities of factor intensities. How does it compare with the previous results?

2.4 Draw a two-panel diagram with ω on the common vertical axis. The horizontal axis of the right panel is w, and that of the left panel is r. Show all the boundary points. Is this diagram affected by the factor intensity assumption?

2.5 Draw a four-panel production diagram where the excess demand replaces the supply and the restricted demand in panel III

2.6 Add to 2.5 another panel (V) underneath panel IV, with the vertical axis denoting the unit profit, $\pi = (p^d - p^s)$. Draw the unit profit against y_2. Indicate the equilibrium point

2.7 Derive graphically the restricted demand curve for an open economy when the world price is allowed to be outside the range of admissible domestic prices, that is, $\overline{p} < p^*$, or $p^* < p$

2.8 Show that the factor measure of factor intensity, $k_1(\omega) < k_2(\omega)$ is equivalent to $\rho < \ell$ where $\rho = K_1/K$ and $\ell = L_1/L$